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Parametrization of the Hungarian Stereographic Map Sheets

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Abstract: A topographic map series of Northern Transylvania was drawn in the 1940s. Georeferencing them needs information about their coordinate systems. These maps used two reference systems known as the Budapest and the Marosvásárhely Stereographic projections. The datum transformation parameters of the former had already been determined in previous studies, but they had to be adjusted slightly to use a common Ferro–Greenwich difference for both systems. Parameters of the Marosvásárhely system were only available as a Molodenskiy transformation determined with insufficient accuracy. In this study, we recalculate the Molodenskiy parameters with higher accuracy and determine six parameters from the usual 7-parameter Helmert (aka. Burša–Wolf) transformation. The ideas given in this paper can be useful in other similar cases when the information about base point coordinates on the old system is very limited: We had access only to the ellipsoidal coordinates of the origin and the projected coordinates of one single point. The accuracy of the resulting transformation was evaluated by aligning the old map sheets with recent base maps.

Keywords: Datum transformation, georeferencing, PROJ.4 string

1. History of the Hungarian stereographic projection

The Hungarian stereographic projection was introduced around 1860. This is the oldest double conformal projection in the world (Hazay, 1967). The mapping uses a conformal auxiliary sphere between the Bessel ellipsoid and the plane. The auxiliary sphere (hereafter Gaussian sphere) is then mapped using the oblique stereographic projection. Thus, its equations are essentially identical to the RD New system used now in the Netherlands (van Hees, 2006), with slight differences detailed in the following.

The Gaussian sphere can have one distortion-free parallel. Usually this is same as the latitude of the origin (this is the case with system RD New), but theoretically it can be parameterized independently. Because the original plan was to use a common Gaussian sphere for the whole territory of the Habsburg Empire, while marking different origins of the stereographic projections for each member state, the standard parallel of the Gaussian sphere is not at the latitude of origin (Hazay, 1967), but at the central latitude of the Habsburg Empire (46°30’ on the Gaussian sphere). Nevertheless, only two states used this system: Hungary and Transylvania (the latter was separated from Hungary between 1849 and 1867, this is the reason for the independent triangulation). The effect of the different placement of the standard parallel and the origin will be discussed later.

Map sheets produced before the stereographic system were mapped in the Cassini–Soldner projection. Its distortions grow gradually depending on the distance from the mid-meridian. Thus, the origins do not have to be at the centre of the area, it is optimal enough if they lie near the central longitude of the area. On the other hand, the distortion of the stereographic projection grows radially with respect to the distance from the origin. Therefore, it is crucial, that the origin must have a central placement.

In the case of Hungary, the traditional origin was the observatory of Gellérthegy, Budapest, which was almost central, so it was retained. However, the origin of the Transylvanian sheets were at Vízakna, which lies near the (former) southern border. Therefore, a new origin had to be chosen at Kesztej-hegy (now Delaul Câștii) near Marosvásárhely (now Târgu Mureș, part of Romania). There was no astronomical observatory at this place, the base point is only marked by a stone.

The coordinate system of the stereographic sheets of Hungary is now called as the Budapest system while the Transylvanian ones as the Marosvásárhely system.

As astronomical and geodesic measurements had already had a long tradition at the Gellérthegy base point, the latitude and longitude at the origin of the Budapest system had been known before the construction of the system. It was calculated as follows: The astronomical coordinates for the Vienna observatory were accepted as coordinates on the Wallbeck ellipsoid. Then, the distance and azimuth to Gellérthegy were measured by triangulation. Thus, coordinates were obtained on the Wallbeck ellipsoid. However, at this time, geodesists suddenly switched to the Bessel ellipsoid without repeating the calculation (Homoródi, 1953). Therefore, the Budapest system is rotated by ca. 12-13 arcseconds compared to the true North. The large values of the rotation parameters in the datum definition will reflect this error.

Ellipsoidal coordinates of other base points were not calculated, but measured distances and angles were corrected for the distortions of the stereographic projection (the distortions of the Gaussian sphere were neglected), and calculated only on the plane. As the linear scale and the arc-to-chord corrections of the stereographic projection can be reformulated to depend solely on the planar coordinates, the latitude of the origin was only used to determine the meridian convergence and the longitude was not used at all. This shows that if we change the longitude of the origin after the measurements, planar coordinates will not change at all, if we slightly change the latitude of the origin, the meridian convergence will slightly change, but the change in planar coordinates will be maximum at millimetre order, so it can be neglected.

The previous investigations show that there was no need for exact coordinates at the origin to start measuring in a stereographic system. Therefore, at the new base point Kesztej-hegy only an approximate astronomical latitude and defining azimuth were measured, and were provisionally accepted as quantities on the Bessel ellipsoid. The longitude of the origin was not even measured (Fasching, 1909). Only plane coordinates were calculated for other points. Later accurate measurements showed that the approximate defining azimuth at the Kesztej-hegy point contained ca. 16 arc-seconds of error, which will result in large rotation parameters when fitting the system to WGS84.

Stereographic coordinates were used only for cadastral purposes until 1920. Topographic maps produced after this year switched to this system, but arbitrarily changed the ellipsoidal coordinates of the origin. As discussed before, this did not change planar coordinates of points, but did change ellipsoidal ones. As this was the first map series that included ellipsoidal coordinates at the map frame, these newly accepted coordinates must be used when georeferencing the map sheets using its coordinate lines. On most (but not all) sheets, a false easting and northing of 500 km was added to both coordinates, so one must proceed with caution.

The Marosvásárhely system was not used between 1920 and 1940, as Transylvania was part of Romania. However, the northern part was returned to Hungary, so the Marosvásárhely system was reintroduced, but with a false easting and northing of 600 km. The 1 : 50 000 maps produced in Hungary used either the Budapest or the Marosvásárhely system, but the sheets were unified and the two systems can be connected at the boundary of the sheets. Our aim is to georeference this map series.

The map series used the Ferro Prime meridian defined to be at exactly 20° West of Paris. We could use any arbitrary value for the Ferro to Greenwich correction, if the datum definition is changed accordingly, and we use that value consistently. Unfortunately, Timár et al. (2003, 2007) used different values for the Budapest and Marosvásárhely systems. For the latter, they apparently used the Albrecht difference, but some other, arbitrary value for the Budapest system. In this paper, Ferro longitudes will be converted using the Albrecht difference: 17°39’46.02”, which was officially accepted in Hungary at that time (Timár, 2007).

2. Parameters of the Budapest system

The original coordinates measured for the point Gellérthegy between 1961 and 1963 are $\Phi = 47°29'14.93''$ and $\Lambda = 36°42'51.69''$ from Ferro on the Wallbeck ellipsoid (Homoródi, 1953). However, as mentioned previously, these coordinates were used only for calculating the meridian convergence, and the triangulation was compensated solely on the plane. Furthermore, these coordinates were considered arbitrarily on the Bessel ellipsoid.

More exact coordinates were assigned to the point a posteriori. Fasching (1909) calculated $\varPhi = 47°29'9.6388''$ and $\varLambda = 36°42'53.5733''$ from Ferro, i.e., 19°3’7.5533” from Greenwich. This did not change the planar coordinates calculated before, but the ellipsoidal coordinates refer to this origin, as they were all computed after this switch. The datum with these parameters is usually referred to as HD1863 in Hungarian literature.

Fasching (1909) also discovered the rotation of the Budapest system described in section 1, but his value contains 6-7” error due to a significant human error during the measurement of the closest Laplace point (Homoródi, 1953). Fortunately, Fasching’s correction of the azimuths was not applied (it would have changed planar coordinates significantly), so the error of the Laplace point is reflected only by the longitude of the origin.

The latitude of the origin is not a standard parallel on the Gaussian sphere. However, this will not result in significant changes in the planar coordinates. The choice of the standard parallel does not influence directions and distances on the Gaussian sphere notably, as its distortions are quite low. However, the radius R of the sphere may change significantly. On the other hand, the radius function of the stereographic projection is $\varrho = 2R\tan(s/2R)$, where s is the spherical distance from the metapole. Due to the small angles, the tangent is almost linear, so the multiplication and the division by R effectively cancels each other. Therefore, we can approximate this map projection by the reparametrization of the similar Dutch system with millimetre accuracy.

Using some base points measured also in recent reference systems, Timár et al. (2003) computed the 7-parameter Helmert transformation between the Hungarian realization of the Bessel ellipsoid. The error of the approximation is ca. 2, max. 5 metres according to Timár. This difference is probably mostly caused by the inaccuracies of the triangulation. Timár used Fasching’s corrected latitude for the calculation, but the longitude is different (19°2’56.9441”). The 10,6092” difference must be added to the rotation around axis Z of the *ECEF* (Earth-centered, Earth-fixed) coordinates. (I.e., we used $rz = +5.0121$ instead of $rz = -5.5971$.) Furthermore, Timár gives the parameters inconsistently in his publications. We will refer to the values found in (Timár, 2008).

Summarizing the previous investigations, the sheets of the Budapest system can be georeferenced using the PROJ.4 string:


```
+proj=sterea +lat_0=47d29'9.6380"
+lon_0=19d3'7.5533" +k=1 +x_0=500000
+y_0=500000 +ellps=bessel
+towgs84=595.75,121.09,515.50,8.2270,
-1.5193,-5.0121,-2.6729 +units=m
+no_defs
```

3. Molodensky parameters of the Marosvásárhely system

As stated in section 1, the coordinates of the origin were not measured before the triangulation. The measurements used a provisional latitude and starting azimuth, triangulation was compensated independently of the Budapest system, but used the same base lines and same ellipsoid. The provisional astronomical latitude of the origin was 46°33'8.85". Later accurate measurements corrected this astronomical latitude to 46°33'9.12" (Fasching, 1909). Although this more accurate latitude was in actual use, Timár et al. (2007) referenced their calculations to the provisional one.

However, there were no direct measurements regarding the longitude. The first attempt to determine the longitude was elaborated by Fasching (1909). He connected the two systems by three points with coordinates known in both systems. With this, he effectively connected the two reference frames. Although computation was carried on the Gaussian sphere, Fasching also gave ellipsoidal coordinates on the Bessel ellipsoid: Φ = 46°33'6.4273" and Λ = 42°3'20.9550" from Ferro, i.e., 24°23'34.9350" from Greenwich.

This is the only calculation that orders a longitude to the base point at Kesztej-hegy. Therefore, Timár's datum parameters used a provisional astronomical latitude together with an ellipsoidal longitude, which is theoretically inconsistent. We should note, however, that this did not affect their results, as it was compensated by the Molodenskiy parameters, and the latitude of origin does not influence the planar coordinates significantly.

For the Marosvásárhely system, we had no control points referring to WGS84, but we needed to recalculate the Molodenskiy parameters, so that we can switch the latitude of origin from the provisional one to the ellipsoidal one. The following idea may be useful for others trying to connect old datums to the WGS84 without access to point coordinates:

1) We identified the location of the base point at the digital topographic map of Romania provided by the Romanian authorities (ANCPI geoportal, TopRo50 database, http://www.geoportal.gov.ro).
2) We measured the Stereo70 coordinates of the point in QGIS with 1 mm accuracy (X = 453190.734 and Y = 561657.545), and read the Baltic height (523.927 m) from the attribute table.
3) We transformed this point from S42 to ETRS89 via the public free service ROMPOS (http://www.rompos.ro). We obtained Φ = 46°33'10.47394" and Λ = 24°23'16.46260" from Greenwich. Accuracy is 2–3 cm. We assumed that ETRS89 and WGS84 are equivalent within the scope of this study.
4) Using the quasi-geoid model of ANCPI, we obtained the ellipsoidal height of the point (563.133 m). The geoid height above HD1863 was neglected.

Molodenskiy parameters were obtained as the difference in ECEF coordinates of the point in the HD1863 and WGS84 systems. Using the notation:

$$\nu = \frac{a}{\sqrt{1 - e^2 \sin^2 \Phi}}$$

We may calculate the ECEF coordinates as:

$$\begin{aligned} x &= (\nu + h) \cos\Phi \cos\Lambda \\ y &= (\nu + h) \cos\Phi \sin\Lambda \\ z &= [\nu(1 - e^2) + h] \sin\Phi \end{aligned}$$

The differences are dx = +589.81, dy = −164.65, dz = +580.45. For now, we have the following PROJ.4 string:

```
+proj=sterea +lat_0=46d33'6.4273"
+lon_0=24d23'34.9350" +k=1 +x_0=600000
+y_0=600000 +ellps=bessel
+towgs84=589.81,-164.65,580.45
+units=m +no_defs
```

4. Helmert transformation of the Marosvásárhely system

As mentioned before, the defining azimuth of the Marosvásárhely system was only provisional at the triangulation. However, we found no evidence that this "provisional" azimuth was ever changed in the system. Therefore, the axes of the coordinate system are not aligned exactly to the true North. Therefore, the accuracy of a simple Molodensky transformation is not sufficient, the effect of this rotation has to be considered.

The literature agrees that the defining azimuth was measured in the direction of Tiglamor (we identified it with the hill Țigla Moruțului near Sărmașu). Nevertheless, its value varies significantly. For example, Mugnier (2017) says that it was 326°57'41.052". (We added 180° to the original values adhering to the current convention of measuring the azimuth from North.) Contrary, we found in Fasching (1909) that this azimuth is valid on datum HD1909, which was never introduced in Transylvania. The other frequently found value (326°57'38.84"), e.g., in (van de Sande, 1910), was surely not used as it was measured more than 20 years after the triangulation (in 1891). Fortunately, Oltay (1914) lists the stereographic coordinates of Tiglamor in Viennese fathoms (X = 7463.794 and Y = −11474.076). From this, we may reconstruct that the actual defining azimuth was 326°57'22.76".

To calculate the same azimuth on WGS84, we obtained the Stereo70 coordinates of the point from the geodatabase of ANCPI mentioned earlier (apparently, this point still exists in recent triangulations), and transformed to ETRS89 using ROMPOS. This yielded Φ = 46°44'54.75012" and Λ = 24°12'09.63184". Solving the second fundamental task of geodesy by the method of Karney (2013) on

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WGS84, we obtained azimuth 326°57’45.117”, thus the Marosvásárhely system is rotated around Kesztej-hegy by $\alpha = 22.357''$ compared to WGS84.

Therefore, we shall do the following transformation:

1) We recenter the three-dimensional Cartesian system to Kesztej-hegy (we subtract its ECEF coordinates on Bessel ellipsoid from the coordinates of other points).
2) We rotate the system by angle α around the normal of the ellipsoid at Kesztej-hegy. The rotation matrix would be very complicated, but as α is very small, we simplified it using the approximations $\sin\alpha \approx \alpha$ and $\cos\alpha \approx 1$.
3) We add the ECEF coordinates of Kesztej-hegy, so that the coordinate system is again Earth-centered, but rotated.
4) We apply the Molodenskiy transformation described in section 3 to gain WGS84 coordinates.

Steps 1)–3) can be described as:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \begin{pmatrix} 1 & \alpha \sin\Phi & -\alpha\cos\Phi\sin\Lambda \\ -\alpha\sin\Phi & 1 & \alpha\cos\Phi\cos\Lambda \\ \alpha\cos\Phi\sin\Lambda & -\alpha\cos\Phi\cos\Lambda & 1 \end{pmatrix} \cdot \left[\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} \right]$$

However, the Helmert (aka. Burša–Wolf) transformation built in common GIS software can only do (using the “position vector” convention):

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} + \begin{pmatrix} 1 & -rz & ry \\ rz & 1 & -rx \\ -ry & rx & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Equating the two transformations with each other, we yield a linear system of equations. The solution is (α must be substituted in radians; ν, e^2, Φ, and Λ are measured on the Bessel ellipsoid):

$$\begin{aligned} dx &= -\alpha\nu e^2 \sin\Phi\cos\Phi\sin\Lambda \\ dy &= +\alpha\nu e^2 \sin\Phi\cos\Phi\cos\Lambda \\ dz &= 0 \\ rx &= -\alpha\cos\Phi\cos\Lambda \\ ry &= -\alpha\cos\Phi\sin\Lambda \\ rx &= -\alpha\sin\Phi \end{aligned}$$

Due to step 4), the Molodenskiy parameters calculated in section 3 must be added to the translations dx, dy, and dz. Rotations rx, ry, and rz should be converted from radians to arc-seconds. The final PROJ.4 string is:

```
+proj=sterea +lat_0=46d33'6.4273"
+lon_0=24d23'34.9350" +k=1 +x_0=600000
+y_0=600000 +ellps=bessel
+towgs84=588.85,-162.55,580.45,
-14.002,-6.350,-16.231,0 +units=m
+no_defs
```

We should note that the seventh parameter (which corresponds to the scale error) cannot be determined from our data, as it needs point coordinates far from the origin. Fortunately, this parameter has very low influence on the *horizontal* position of the points, so we left it at zero.

5. Checking the accuracy of the transformation

The accuracy of the transformation was affirmed by georeferencing the map series created in the 1940s, and comparing their geometry to the current base map of Romania (TopRo50). From the latter only basic layers (triangulation points, roads, rivers, settlements, etc.) were extracted, and are displayed in harsh colours (mostly red and orange) in the figures. We first georeferenced the map sheets using the Moldensky parameters described by Timár et al. (2007). We were surprised, as the method of Timár should theoretically give an almost perfect alignment at the origin of the projection, but we observed ca. 30 m (!) offset at Kesztej-hegy (fig. 1).

Figure 1. Even the origin (Kesztej-hegy) is misaligned using the parameters of Timár et al. (2007) (left), it is aligned perfectly in this study (right). Red symbols indicate the TopRo50 database.

The results of this study are mostly useful at areas far from the origin, where the rotation of the axes affect the projected coordinates. Fig. 2 displays the area of village Kézdiszentlélek (Sânzieni), which is 150 km from the origin. The streets are now aligned perfectly even here. The misplacement of the point Orotvány-tető was 60 m with Timár’s parameters, which is much lower accuracy than he aimed!

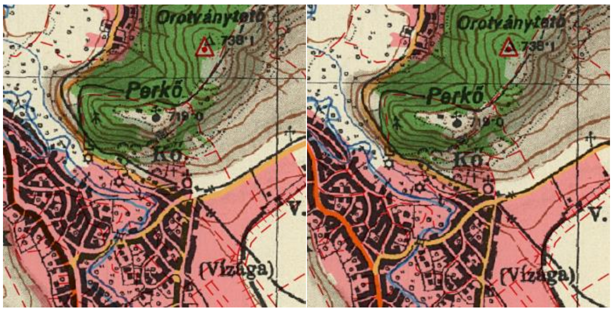

Figure 2. The alignment was poor far from the origin (left), is now close to perfect (right)

Measuring the misplacement of triangulation points, road crossings, and railroad features throughout Northern Transylvania, the error was usually below 15 m (0.3 mm in the map scale) using the parameters listed here. As such small difference can also be the result of cartographic generalization, we cannot give the accuracy of the transformation, but it is presumably better than 15 m, which is acceptable for usual GIS purposes. We may for example track changes of the landscape since the 1940s (fig. 3).

Figure 3. Roads are aligned perfectly, but river Szamos (Someş) changed its course

Apparently the triangulation bases of the system were imperfect, as there are a few significantly misaligned parts. The worst example is the town of Bánffyhunyad (Huedin), which is still misplaced by 90–95 m. We assume that it is a result of a local error in the past triangulation, as all other parts of the same sheet align much better, e.g., the misalignment of Nagysebes (Valea Drăganului) just 15 km away was measured below 5 m.

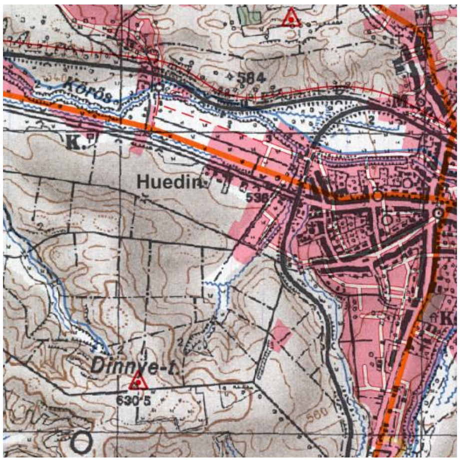

Figure 4. No alignment could be achieved in some areas

That being said, the transformation given in this paper is appropriate for georeferencing at most parts of Transylvania with an accuracy of 5–15 m. Outlier areas are probably an error of the original map, and could be improved only by, e.g., a local grid shift.

6. Further plans

The deterministic nature of the calculation presented in this paper does not make it possible to consider the inner measurement errors of the triangulation. Although theoretically the parameters presented here should perfectly match the old systems, probably a very different set of parameters could approximate its actual placement better. Therefore, we suspect that if more point coordinates had been available, a least-squares fitting method would have resulted in better accuracy. The authors would like to obtain more data to calculate parameters using this method.

Furthermore, the scale error of the system could not be estimated due to the insufficient amount of data. A later study should provide at least an approximate value for this.

7. Acknowledgements

The database TopRo50 used for the calculation and for the base map of the figures was provided by the ANCPI under the following licence agreement: http://geoportal.ancpi.ro/portal/sharing/rest/content/items/94ebfdce60b34da7958216b1558e781d/data

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